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PHYSICS.—*Avogadro's constant and atmospheric transparency.* F. E. FOWLE, Astrophysical Observatory, Smithsonian Institution.[1]

In this communication are given coefficients for the transmissibility of radiation through the dry air vertically above Mount Wilson together with factors and a formula for computing the transmissibility for moist air for other zenith distances and altitudes above sea-level where dust has become a negligable quantity (above 1000 meters). No account is, however, taken in this formula of selective absorption.

By means of Rayleigh's formula connecting the scattering of light passing through a gas with the number of molecules, these coefficients have been used to compute the number of molecules under standard conditions. The merits of the present reductions over those of an earlier paper by the author lie in using the index of refraction proper to each wave-length in place of a mean value, in better values for the transmission coefficients and in a more accurate value for the barometric pressure, the value before used being taken at an appreciable higher altitude on the mountain than our observatory. The mean results give for Loschmidt's number or the number of molecules in a cubic centimeter of a gas at 0°C. and 76 cm. pressure

$$n_0 = (2.70 \pm 0.02) \times 10^{19}$$

[1] This paper, read before the Philosophical Society, October 10, 1914, will appear in full elsewhere, probably in the Astrophysical Journal.

and for Avogadro's constant or the number of molecules per gram molecule

$$N = (6.05 \pm 0.04) \times 10^{23}.$$

The agreement of the above value for n_0 with what is perhaps the best value from other methods, $(2.705 \pm 0.005) \times 10^{19}$ (Millikan), must give weight to the accuracy of the estimation of the atmospheric losses in the determinations of the solar radiation by the Smithsonian Observatory. Some criticism is made of the procedure of L. V. King who uses our observed moist-air transmission coefficients for a similar purpose, mainly in that he uses coefficients for wave-lengths where selective absorption is present and for such losses Rayleigh's formula does not hold.

A remnant of the volcanic dust from the eruption of Mount Katmai, Alaska, in 1912, scattering somewhat less than 3 per cent of the incident radiation, is indicated by the 1913 transmission coefficients. It is perhaps worth noting that, fine as this dust must be to have remained suspended in the upper air over a year, its scattering of radiation scarcely varies with the wave-length, at least between the limits 0.38μ and 0.81μ.

PHYSICS.—*Measurement of the extraordinary refractive index of a uniaxial crystal by observations in convergent light on a plate normal to the optic axis.* H. E. MERWIN, Geophysical Laboratory.

During an investigation of the optical properties of the trioxides of arsenic and antimony, it was found that attempts to crush the tabular, hexagonal crystals into grains which could be suitably oriented for the determination of the extraordinary refractive index, resulted in producing aggregates, probably due to gliding. The possibility of measuring ϵ by a study of the interference rings on basal plates in convergent polarized light was then considered.

The figure represents a vertical section through a basal plate of an optically negative uniaxial crystal. The wave-normals for the ordinary and the extraordinary wave originating from the same incident wave are shown. They form with the optic axis

the angles α and α_1, within the crystal, and the angle β in the air where they are parallel. The emergent waves are supposed to totally interfere in one of the observed dark rings. In r and s, between normals from the surface, there are the same number of waves, then the difference in phase between the two waves entering the microscope is expressed by the equation $\frac{\omega m}{\lambda} - l = \frac{m_1 \epsilon_1}{\lambda}$, in which l is the number of wavelengths that the extraordinary wave is in advance of the ordinary wave as determined by the serial number of the dark ring, counted outward from the optic axis; λ is the wave-length in air; ω the ordinary refractive index; and ϵ_1 the refractive index of the extraordinary wave whose normal makes the angle α_1 with optic axis. e is the thickness of the section.

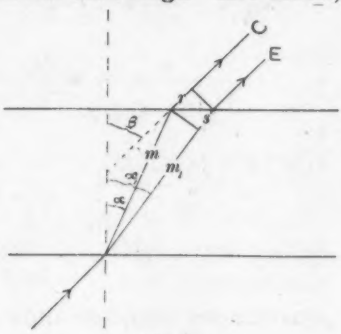

Fig. 1

$$\sin \alpha = \frac{\sin \beta}{\omega}, \text{ and } m = \frac{e}{\cos \alpha}$$

Then $m = \dfrac{e}{\sqrt{1 - \dfrac{\sin^2 \beta}{\omega^2}}}$

Now $m_1 = m \cos (\alpha_1 - \alpha) = m (\cos \alpha \cdot \cos \alpha_1 + \sin \alpha \cdot \sin \alpha_1) \ldots\ldots (1)$

Then $\frac{\omega m}{\lambda} - l = e_1 m (\cos \alpha \cdot \cos \alpha_1 + \sin \alpha \cdot \sin \alpha_1)$ for a negative crystal. For a positive crystal l is positive, therefore in the general case, after substituting

$$\frac{\omega e}{\lambda \cos \alpha} \pm l = \frac{\epsilon_1 e (\cos \alpha \cdot \cos \alpha_1 + \sin \alpha \cdot \sin \alpha_1)}{\lambda \cos \alpha} =$$

$$\frac{\epsilon_1 e}{\lambda} \cos \alpha_1 + \sin \alpha_1 \frac{\sin \alpha}{\cos \alpha} \ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\omega e}{\lambda\sqrt{1-\frac{\sin^2\beta}{\omega^2}}} \pm l = \frac{\epsilon_1 e}{\lambda}\left(\sqrt{1-\frac{\sin^2\beta}{\epsilon_1^2}} + \frac{\frac{\sin^2\beta}{\epsilon_1\omega}}{\sqrt{1-\frac{\sin^2\beta}{\omega^2}}}\right)$$

$$\frac{\frac{\omega}{\sqrt{\omega^2-\sin^2\beta}}}{\frac{1}{\omega}} \pm \frac{l\lambda}{e} = \sqrt{\epsilon_1^2-\sin^2\beta} + \frac{\sin^2\beta}{\sqrt{\omega^2-\sin^2\beta}}$$

$$\sqrt{\epsilon_1^2-\sin^2\beta} = \frac{\omega^2}{\sqrt{\omega^2-\sin^2\beta}} + \frac{\sin^2\beta}{\sqrt{\omega^2-\sin^2\beta}} \pm \frac{l\lambda}{e}$$

$$\epsilon_1^2 = \omega^2 + \left(\frac{l\lambda}{e}\right)^2 + \frac{2l\lambda}{e}\sqrt{\omega^2-\sin^2\beta} \quad \ldots\ldots\ldots\ldots (3)$$

A simpler equation than (3), which for such angles as are measurable by the microscope contains an entirely negligible error not greater than 2 or 3 units in the 5th decimal place, is derived as follows: assume that the cosine of the very small angle $(\alpha_1 - \alpha)$ equals 1, then in (1) $m = m_1$, and in (2)

$$\frac{\omega e}{\lambda\cos\alpha} \pm l = \frac{\epsilon_1 e}{\lambda\cos\alpha} \text{ or } \epsilon_1 = \omega \pm \frac{l\lambda\cos\alpha}{e} = \omega \pm \frac{l\lambda}{e}\sqrt{1-\frac{\sin^2\beta}{\omega^2}} \quad (3a)$$

Equation (3) gives exactly, and equation (3a) almost exactly the refractive index of the extraordinary wave whose normal makes with the optic axis the angle α_1 within the crystal, and the angle β in the air.

From the general equation of the index ellipsoid

$$\epsilon = \frac{\omega\sin\alpha_1}{\sqrt{\frac{\omega^2}{\epsilon_1^2}-\cos^2\alpha_1}} = \frac{\frac{\omega\sin\beta}{\epsilon_1}}{\frac{1}{\epsilon_1}\sqrt{\frac{\omega^2}{\epsilon_1^2}-\left(1-\frac{\sin^2\beta}{\epsilon_1^2}\right)}}$$

$$\epsilon = \frac{\omega\sin\beta}{\sqrt{\omega^2-\epsilon_1^2+\sin^2\beta}} \quad \ldots\ldots\ldots\ldots (4)$$

Substituting[1] (3)

[1]Substituting (3a) we have $\epsilon = \dfrac{\omega\sin\beta}{\sqrt{\omega^2-\left(\omega \pm \frac{l\lambda}{e}\sqrt{1-\frac{\sin^2\beta}{\omega^2}}\right)^2+\sin^2\beta}}$. (6a)

This value of ϵ, though not precisely correct, has no measurable error.

$$\epsilon = \frac{\omega \sin \beta}{\sqrt{\omega^2 - \left[\omega^2 + \left(\frac{l\lambda}{e}\right)^2 \pm \frac{2l\lambda\omega}{e}\sqrt{1 - \frac{\sin^2\beta}{\omega^2}}\right] + \sin^2\beta}} \quad \ldots (5)$$

$$\epsilon = \frac{\sin \beta}{\sqrt{1 - \left(\sqrt{1 - \frac{\sin^2\beta}{\omega^2}} \pm \frac{l\lambda}{e\omega}\right)^2}} \quad \ldots\ldots\ldots (6)$$

This is a true equation for the extraordinary refractive index of a uniaxial crystal.[2]

From measurements of β and l on two rings of the same section, equations can be formed from (6) which when solved simultaneously permit the elimination of one of the constants ω, ϵ, λ, e. However, the resulting equations have no practical application, for very slight errors in the observed values of β may make very large errors in the values calculated.

If ω, ϵ and λ are known e, the thickness of the plate, is readily and accurately found thus:

By transposition (4) becomes $\epsilon_1 = \sqrt{\omega^2 + \sin^2\beta\left(1 - \frac{\omega^2}{\epsilon^2}\right)}$

Then by substituting (3a)

$$\omega \pm \frac{l\lambda}{e}\sqrt{1 - \frac{\sin^2\beta}{\omega^2}} = \sqrt{\omega^2 + \sin^2\beta\left(1 - \frac{\omega^2}{\epsilon^2}\right)}$$

From which $e = \dfrac{l\lambda\sqrt{1 - \frac{\sin^2\beta}{\omega^2}}}{\sqrt{\omega^2 + \sin^2\beta\left(1 - \frac{\omega^2}{\epsilon^2}\right)} - \omega}$

From the idea of the isochromatic surface several roughly approximate formulas[3] have been developed which contain a term in which the double refraction appears. These assume

[2] This equation is identical with equation g (expressed in the form of equation 6) of the article by F. E. Wright in this issue; the present equation is derived, however, by a different method.

[3] See Preston's The Theory of Light, pp. 393–345; and Duparc and Pearce's Traité de Technique Mineralogique et Pétrographique, pp. 312–313.

that the angle α within the crystal is known. This angle cannot be obtained from microscopic measurements unless one of the indices of refraction is known.

For measuring the dark rings a standardized microscope with screw micrometer ocular is most satisfactory, but a cross-grating ocular can be used effectively, especially if, when the rings are closely spaced, the readings are made along the diagonals of the squares. The average of readings in the four quadrants should be taken. If this is done, errors due to distortion by the tube nicol and to lack of precise perpendicularity of the plate to the optic axis are largely compensated. If the plate lacks 5° of being normal to the optic axis no appreciable error will result.

As shown by measurements on several plates of arsenic triiodide and calcite the angle β can be found within about 0.2°. The corresponding calculated value of $\omega - \epsilon$ is in error 2 per cent when $\beta = 10°$, 1 per cent when $\beta = 20°$, and $\frac{7}{10}$ per cent when $\beta = 35°$.

PHYSICS.—*Measurements of refractive indices on the principal optical sections of birefracting minerals in convergent polarized light.* FRED. EUGENE WRIGHT, Geophysical Laboratory.

The principal optical sections of birefracting minerals play an important rôle in petrographic microscope work. They are parallel to the planes of symmetry of the optic ellipsoid and exhibit, in convergent polarized light, characteristic interference figures. On any given principal optical section two of the three principal refractive indices of the mineral can be measured directly by the immersion method. The third principal refractive index of the mineral can also be determined by use of the phenomena observed on the same section in convergent polarized light. The methods available for this determination involve some computation and have not been used to any extent by petrologists; but, in exceptional cases, especially on thin tabular crystals, the usual methods may prove inadequate and the methods outlined below may then be used to advantage to obtain accurate results. The usefulness of such formulas has

recently been shown by H. E. Merwin, of this Laboratory, in the measurement of the refractive indices of minute uniaxial crystal flakes of arsenic iodide; Dr. Merwin measured the angular width of the concentric interference rings and from these computed the refractive index ϵ by means of a formula[1] which he found to furnish accurate results for uniaxial sections normal to the axis.

Derivation of formulae.[2] For the particular case of principal sections the accurate formulae can be derived directly as follows: Let figure 1 represent a principal section of a birefracting plate. Let KC be the incident, plane-polarized beam of light (in air); CA and CB, the two refracted wave normals; AD and BE the two parallel emergent beams (in air), which enter the objective and are brought to focus in its upper focal plane whence they pass through the analyzer to the eye of the observer. For the incident and refracted wave normals the general relations (invariants) obtain

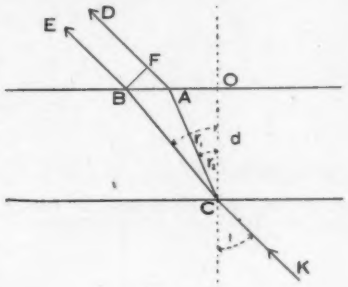

Fig. 1

$$n_0 \sin i = r_1 \sin r_1 = n_2 \sin r_2 \ldots\ldots\ldots\ldots (1)$$

in which i is the angle of incidence; r_1, the angle of refraction of the faster wave; r_2, the angle of refraction of the slower wave; n_0, the refractive index of air (for present purposes n_0 may be considered = 1); n_1, the refractive index of the faster wave; n_2, the refractive index of the slower wave. The interference phenomena are the result, primarily, of the difference in optical path between the two refracted waves, starting at C and attaining again a common wave front at BF. This path-difference is expressed by the equation:

$$\Delta = k \cdot \lambda = n_2 \cdot CA + n_0 \cdot AF - n_1 \cdot CB$$

[1] J. Wash. Acad. Sci., **4**: 532, equation (6a). 1914.

[2] The derivation of the formulae for the general case of any section of a birefracting mineral is given in text books on crystal optics (Pockels, Lehrbuch der Kristalloptik. Chaps. II, IV, IX, 1906).

or

$$k\lambda = n_2 \cdot \frac{d}{\cos r_2} + n_0 \cdot d\,(\tan r_1 - \tan r_2)\,\sin i - n_1 \cdot \frac{d}{\cos r_1} \quad ..(2)$$

in which Δ is the path difference; λ, the wave length of the interfering bands; k, the number of wave lengths included in the path-difference Δ. From (1) and (2) we obtain the relation:

$$\frac{k \cdot \lambda}{d} = n_2 \cos r_2 - n_1 \cos r_1 \quad \ldots\ldots\ldots\ldots\ldots (3)$$

$$\frac{k \cdot \lambda}{d} = \frac{n_2}{\frac{CA}{d}} - \frac{n_1}{\frac{CB}{d}} \quad \ldots\ldots\ldots\ldots\ldots (3a)$$

an equation which in the form (3a) states that the path-difference for unit thickness of plate is equal to the difference between

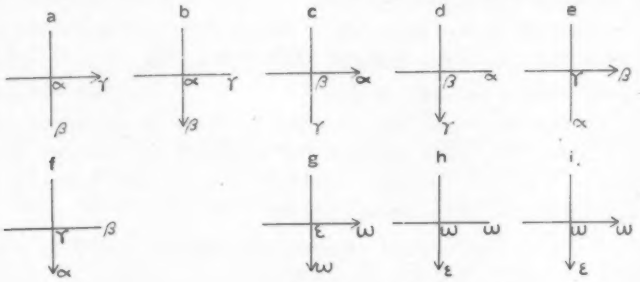

Fig. 2

the reciprocals, referred to air, of the paths CA and CB of the two refracted wave normals.

Values for the terms $n_1 \cos r_1$, and $n_2 \cos r_2$ in equation (3), can be deduced from the general equation of the index surface referred to the principal axes

$$\frac{\nu_1^2}{\frac{1}{\alpha^2} - \frac{1}{n^2}} + \frac{\nu_2^2}{\frac{1}{\beta^2} - \frac{1}{n^2}} + \frac{\nu_3^2}{\frac{1}{\gamma^2} - \frac{1}{n^2}} = 0 \quad \ldots\ldots\ldots\ldots (4)$$

in which ν_1, ν_2, ν_3 are the direction cosines of the refracted wave normal, n, its refractive index and α, β, γ, the principal refractive indices of the mineral.

To apply these formulas to a particular case, let us consider a plate to be cut normal to α and the plane of incidence to be the $\alpha\,\gamma$ plane (fig. 2a). Under these conditions

$$n_1 = \beta,\ r_1 = r_\beta;\ \nu_1 = \cos r_2,\ \nu_2 = 0,\ \nu_3 = \sin \gamma_3;$$

and equation (4) becomes

$$\frac{\cos^2 r_2}{\frac{1}{\alpha^2} - \frac{1}{n^2}} + \frac{\sin^2 r_2}{\frac{1}{\gamma^2} - \frac{1}{n^2}} = 0$$

from which we obtain on expansion

$$n_2^2 \cos^2 r_2 = \gamma^2\left(1 - \frac{n_1^2 \sin^2 r_1}{\alpha^2}\right) = \gamma^2\left(1 - \frac{\sin^2 i}{\alpha^2}\right) = \gamma^2 \cos^2 r_\alpha$$

if we consider r_α to be the angle of refraction of the wave which satisfies the equation

$$\sin i = \alpha \cdot \sin r_\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Equation (3) can now be written

$$\frac{k \cdot \lambda}{d} = \gamma \sqrt{1 - \frac{\sin^2 i}{\alpha^2}} - \beta \cos r_\beta \quad \ldots\ldots\ldots\ldots (6)$$

or

$$\frac{k \cdot \lambda}{d} = \gamma \cos r_\alpha - \beta \cos r_\beta \quad \ldots\ldots\ldots\ldots\ldots (6a)$$

For purposes of computation equations (5) and (6a) are more convenient than (6).

The equations for the different cases indicated in figure 2[3] are:

$$\frac{k \cdot \lambda}{d} = \gamma \cos r_\alpha - \beta \cos r_\beta \quad \text{and} \quad \sin i = \alpha \cdot \sin r_\alpha \ldots . a \text{ (fig. 2)}$$

$$-\frac{k \cdot \lambda}{d} = \beta \cos_\alpha \quad - \gamma \cos r_\gamma \quad \text{and} \quad \sin i = \alpha \cdot \sin r_\alpha \ldots . b \text{ (fig. 2)}$$

$$-\frac{k \cdot \lambda}{d} = \alpha \cos r_\beta - \gamma \cos r_\gamma \quad \text{and} \quad \sin i = \beta \cdot \sin r_\beta \ldots . c \text{ (fig. 2)}$$

[3] In each small figure of figure 2 the arrow represents the trace of the plane of incidence on the horizontal plane.

$$\frac{k\cdot\lambda}{d} = \gamma\cos r_\beta - \alpha\cos r_\alpha \quad \text{and} \quad \sin i = \beta\cdot\sin r_\beta \ldots\ldots d \text{ (fig. 2)}$$

$$\frac{k\cdot\lambda}{d} = \beta\cos r_\gamma - \alpha\cos r_\alpha \quad \text{and} \quad \sin i = \gamma\cdot\sin r_\gamma \ldots\ldots e \text{ (fig. 2)}$$

$$-\frac{k\cdot\lambda}{d} = \alpha\cos r_\gamma - \beta\cos r_\beta \quad \text{and} \quad \sin i = \gamma\cdot\sin r_\gamma \ldots\ldots f \text{ (fig. 2)}$$

For uniaxial minerals these equations reduce to the following, in which the positive of the $\pm$ sign should be used for a positive mineral, the negative sign for a negative mineral.

(1) Section normal to the principal axis

$$\pm\frac{k\cdot\lambda}{d} = \omega(\cos r_\epsilon - \cos r_\omega) \quad \text{and} \quad \sin i = \epsilon\cdot\sin r_\epsilon \ldots\ldots g \text{ (fig. 2)}$$

(2) Section parallel to the principal axis

$$\pm\frac{k\cdot\lambda}{d} = (\epsilon - \omega)\cos r_\omega \ldots\ldots h \text{ (fig. 2)}$$

$$\pm\frac{k\cdot\lambda}{d} = \epsilon\cos r_\epsilon - \omega\cos r_\omega \quad \text{and} \quad \sin i = \epsilon\sin r_\epsilon \ldots\ldots i \text{ (fig. 2)}$$

It is of interest to note that in the case of normal incidence ($i = r = 0$) all of the above equations reduce to the ordinary expression for the birefringence $\frac{k\cdot\lambda}{d} = n_2 - n_1$ (i.e., path-difference varies with thickness of plate and with the birefringence.) In the case of a section normal to a bisectrix the path-difference for waves along an optic axis (binormal) is zero and the angle of refraction is half the optic axial angle ($r_\beta = V$); thus from equation *a* we find

$$0 = \gamma\cos r_\alpha - \beta\cos V_\alpha \text{ or}$$

$$\cos V_\alpha = \frac{\gamma}{\beta}\cdot\cos r_\alpha \ldots\ldots (7)$$

from which the usual expression $\cos^2 V_\alpha = \frac{\frac{1}{\alpha^2} - \frac{1}{\beta^2}}{\frac{1}{\alpha^2} - \frac{1}{\gamma^2}}$ can be easily

derived. Equation (6) is a convenient form to use for computing the third refractive index, provided the optic axial angle and two of the refractive indices are known.

Methods of measurement. The above equations express relations between certain quantities which can be measured as follows:

d, thickness of plate, measured either by a micrometer or a spherometer or by one of the standard microscope methods. On thin plates (a few hundredths of a millimeter thick) the error may amount to 10 per cent; on thicker plates the percentage error is correspondingly less. On many sections the thickness is best found by computation from the interference fringes. The exact path-difference, $k\cdot\lambda$, is ascertained by direct count of the interference bands.
n_1, n_2, two of three principal refractive indices of the mineral measured on the given plate either by the immersion method or other standard method. Error should not exceed 0.001.
i, the angle of incidence for the interference line selected, is measured by use of the petrographic microscope equipped with Bertrand lens and either a screw-micrometer ocular or a graduated scale in the eyepiece. On sections of minerals of strong birefringence or on thick plates the interference bands are sharp and the errors of reading should be considerably less than 1°. With thin plates and minerals of medium or weak birefringence the accuracy of the readings is less because of the wide interference lines, but in this case the need for greater accuracy decreases so that on nearly all plates the measurements should furnish refractive index values which are adequate for most purposes.

In certain cases the thickness is difficult to determine; also the order, k, of the interference line may be uncertain, or only one refractive may have been determined. These and other problems can be solved by the measurement of several interference lines. Thus in case the measurement of the thickness is not feasible, we have on a section normal to α for two interference bands, which can be measured, the equation

$$\frac{k_1}{k_2} = \frac{\gamma \cos r_{\alpha 1} - \beta \cos r_{\beta 1}}{\gamma \cos r_{\alpha 2} - \beta \cos r_{\beta 2}} \quad \ldots\ldots\ldots\ldots (8)$$

in which $\cos r_{\alpha 1}$ and $\cos r_{\alpha 2}$ are the unknowns, depending on i_1 and i_2 and α. This equation is most readily solved by assuming a value approximately correct for α; for this value of α, $\cos r_{\alpha 1}$ is computed by means of equation (5). The value of $\cos r_{\alpha 2}$

is then derived from equation (8) and from it and $i_{\alpha 2}$ a second value for α obtained, which will probably be either higher or lower than that originally assumed; a second value for α is accordingly chosen and the computation repeated; the new value of α will probably also be too low or too high. If now the four values thus obtained be plotted on ordinates to scale and a straight line be drawn connecting the assumed values, and a second line connecting the computed values, the intersection of these two lines furnishes a value of α which is sufficiently exact for most purposes. Analogous methods of procedure can be used in solving for two unknown refractive indices, or for path-differences, or for wave length of light employed.

In the special case of a section normal to one of the bisectrices the optic axial angle can be computed with a fair degree of accuracy from measurements on interferences fringes, even though the actual optic axes are outside the field of view. For this purpose, several interference lines are measured to obviate the determination of the thickness of the plate. In case only one interference band is visible the method is still applicable, provided a plate of known path-difference (e.g. $\frac{1}{2}\lambda$) be inserted below the condenser and the path-difference of the interfering waves be increased or decreased a definite fraction of a wave length.

Measurements on crystal plates.

(1) *Uniaxial. Plate normal to axis.* (Equation *g*)

(a) Calcite plate (V. and H. collection No. 22). Optically negative; $d = 0.233$ mm. (computed); $\lambda = 0.000589$ mm.; $\omega = 1.658$; $\epsilon = 1.486$.

k	1	2	3	4	5	6	7	8
i	10°3	15°0	18°4	21°4	24°0	26°5	28°5	30°5
ϵ	1.477	1.485	1.485	1.483	1.486	1.488	1.486	1.485

With the exception of the first value of ϵ in this table (interference band too wide for accurate readings) the computed values of ϵ do not vary greatly and the average value, $\epsilon = 1.4854$, is sufficiently accurate for most purposes.

(b) Zircon plate (V. and H collection No. 26) optically positive; $d = 0.545$ mm. (computed); $\lambda = 0.000589$ mm.; $\omega = 1.930$; $\epsilon = 1.983$.

k	1	2	3
i	26°1	38°8	48°0
ϵ	1.984	1.984	

In this instance the refractive index was computed from equation g for $k = 1$ and $k = 2$ and from equation g for $k = 1$ and $k = 3$. The agreement is satisfactory.

(2) *Uniaxial. Plate parallel to axis.* (Equation h)

Calcite plate (V. and H. collection No. 23.) Optically $-$; $d = 0.082$ mm.; $\lambda = 0.000589$ mm.; $\omega = 1.658$; $\epsilon = 1.486$.

k	24	23	22	21	20	19
i	0	16°.5	23°.5	29°.8	33°.2	37°.5
ϵ	1.486	1.487	1.487	1.485	1.487	1.486

Average $\epsilon = 1.4863$. On this section the path-difference in wave lengths for the different interference lines was ascertained by computation, the plate being too thick for the determinination of the path-differences by ordinary methods of compensation.

(3) *Biaxial. Plate normal to a bisectrix.* (Equations e, f)

Anhydrite. Plate approximately normal to the acute bisectrix γ; $\lambda = 0.000589$ mm.; $\alpha = 1.570$; $\beta = 1.575$; $\gamma = 1.613$.

(a) Plane of incidence $\gamma\,\alpha$ plane. (Equation f)

k	1	0	-1
i	17°.0	35°.3	43°.5

$\gamma = 1.615$ (computed from equations for $k = 1$ and $k = -1$); $\gamma = 1.609$ (computed from equation for $k = 0$).

(b) Plane of incidence $\gamma\,\beta$ plane. (Equation e)

k	2	3	4
i	25°.4	39°.3	48°.5

$\gamma = 1.608$ (computed from equations for $k = 2$ and $k = 3$); $\gamma = 1.607$, (computed from equations for $k = 2$ and $k = 4$).

The values of γ thus obtained vary within relatively large limits. This may be due to the fact that the section was not exactly normal to the bisectrix and also that the values of α and β are slightly in error. In this instance γ is considerably larger than α and β and in computing γ a slight variation in either α or β produces a relatively large variation in γ. The mean of the values listed above is 1.610 or 0.003 less than the refractive index γ given for anhydrite.

(4) *Biaxial. Plate perpendicular to the optic normal.* (Equations c, d)

Selenite. Plate parallel to cleavage plane (010). Normal to β; $d = 0.700$ mm. (measured by micrometer); $\lambda = 0.000589$ mm.; $\alpha = 1.5204$; $\beta = 1.5227$; $\gamma = 1.5296$.

(a) Plane of incidence $\beta\alpha$ plane. (Equation c)

k	11	12
i	27°.4	57°.2
β	1.523	1.523

(b) Plane of incidence $\beta\,\gamma$ plane. (Equation d)

$k = 10$; i = 51°.2; $\beta = 1.522$.

Average $\beta = 1.5227$

In this instance the path-difference, in terms of wave length, for the different interference lines was computed.

These examples suffice to show the order of accuracy of the different methods and also to indicate the variety of problems to which the formulae are applicable. At best, however, these methods are of limited application, although in special cases they render good service.

TECHNICAL PHYSICS.—*The laws of lubrication of horizontal journal bearings.* M. D. HERSEY, Bureau of Standards. Communicated by E. BUCKINGHAM.

I. RELATION OF LAWS OF LUBRICATION TO BEARING DESIGN

1. The two laws of lubrication needed in designing. In the design of horizontal journal bearings, after all questions of strength and rigidity have been disposed of, there usually remains a question as to length and diameter, which must be settled by reference to the laws of lubrication. Evidently too short a bearing is in danger of abrasion, while too long a bearing entails needless dissipation of power.

Let the "coefficient of friction," f, be defined by the equation

$$f = \frac{F}{L} \tag{1}$$

in which F is the frictional resisting force and L the load on the bearing perpendicular to its axis. Let the "bearing pressure," p, be defined by the equation

$$p = \frac{L}{lD} \tag{2}$$

in which l is the length of the bearing and D the diameter of the journal. Let p_0 denote the "carrying power" or greatest permissible bearing pressure. Then the shortest permissible length of a bearing, l_0, may be calculated from the equation

$$l_0 = \frac{L}{D} \cdot \frac{1}{p_0} \tag{3}$$

while the power dissipated in this bearing at a speed of n revolutions per unit time will be

$$P = \pi \, . \, D n \, . \, L \, . f \tag{4}$$

Equations (3) and (4) are purely formal and their practical use demands a knowledge of some relation

$$f = \varphi_1 (p, n, D, l, \text{etc}) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

between the coefficient of friction and all the physical conditions governing the action of lubrication; together with some relation

$$p_0 = \theta_1, (n, D, l, \text{etc.}) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

between carrying power and the various controlling factors. Equations (5) and (6) symbolize the two laws of lubrication which are needed in designing. They may be called the "law of friction" and the "law of carrying power" respectively.

2. Assumptions relating to carrying power. In designing bearings it is of the utmost importance to decide whether carrying power and speed shall be assumed to vary in the same or in opposite directions. The former assumption is the one commonly given in text-books. But the latter assumption appears to represent the practice of the General Electric Company,[1] and will be shown in this paper[2] to be directly deducible from physical facts.

II. THE DYNAMICS OF LUBRICATION

3. Scope of the problem and method of attack. This paper is limited to the consideration of horizontal journal bearings lubricated with clean oil and running under steady conditions. We exclude metallic contact (very heavy load with very low speed), also the case of any appreciable thrust due to forced lubrication. With these restrictions, our problem is to map out the laws of lubrication as completely as can be done on the ground of commonly accepted physical principles. But it will appear that we need not exclude oil-grooves, ring-oiling, eccentric load, worn (i.e. non-circular) bearings, or any other purely geometrical irregularities. Thus, we attack a more general case than could be handled by mathematical reasoning alone.

Under the above limitations we may regard the frictional resistance of a bearing as due entirely to the force required to shear the oil; while the tendency of a journal to center itself at high speeds may be attributed to the wedging action of the oil

[1] Alford, Bearings and their Lubrication (1911), p. 81.

[2] See equation (27).

film as it is dragged under the journal at the point of nearest approach. Let the film thickness at this point be x, while c denotes the mean radial clearance. Call the ratio $\frac{x}{c}$ the relative film thickness. Then *if* it be granted that *all bearings are equally safe when running with the same relative film thickness*, we may conveniently define carrying power as that bearing pressure which reduces the relative film thickness to some prescribed value $\left(\frac{x}{c}\right)_0$. The determination of f and p_0 as indicated in equations (5) and (6) therefore demands a physical analysis of the effect of various conditions on F and x respectively.

4. Physical conditions governing the action of lubrication. Temperature is evidently a governing factor, and will in turn depend in some complicated way on the speed. But temperature can influence friction and film thickness only indirectly, through its effect on the clearance and on the viscosity of the oil. The problem may therefore be simplified by treating the dynamics of the case separately from heating effects.

The dynamical factors on which F and x may depend will evidently include the size and shape of the journal and bearing, which may be specified by D, $\frac{l}{D}$, $\frac{c}{D}$, and such other length-ratios r', r'', etc., as may be needed to fix the shape of the oiling arrangements, deviation from circular section due to wear, etc.; the load, L, and its line of action, specified by length-ratios r''', etc.; the speed of the journal specified by the number of revolutions n, per unit time; the relative oil supply, specified by the ratio, S, of the volume of oil in the bearing to the whole volume of the clearance space; and the mechanical properties of the lubricant, the viscosity μ being usually the only effective property. *If* the foregoing list includes *all* the determining factors we may write symbolically

$$F = \varphi_2\left(n, L, \mu, D, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{7}$$

$$x = \psi_2\left(n, L, \mu, D, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{8}$$

in which r stands for all the length ratios r', r'', r''', etc., and in which φ_2 and ψ_2 are unknown functions.

While the list in question does include all the determining factors met with in *ordinary* cases, it certainly does not include all that might be encountered in extreme cases. For example, with exceptionally high speeds and wide clearances the effects of centrifugal force and of turbulent motion would begin to be felt, thus involving the density of the oil in addition to its viscosity. The question of just where the dividing line comes between "ordinary" and "extreme" cases is an important one but must be passed over in the present abridged report.

5. Derivation of general form of the laws of lubrication by dimensional reasoning. A straight forward application of the principle of dimensional homogeneity[3] to equations (7) and (8) throws them at once into the form

$$F = L \cdot \varphi_3\left(\frac{\mu n D^2}{L}, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{9}$$

and

$$x = c \cdot \psi_3\left(\frac{\mu n D^2}{L}, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{10}$$

Combining (9) and (10) with (1) and (2), then gives

$$f = \varphi\left(\frac{\mu n}{p}, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{11}$$

and

$$\frac{x}{c} = \psi\left(\frac{\mu n}{p}, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{12}$$

or

$$\frac{p}{\mu n} = \theta\left(\frac{x}{c}, \frac{c}{D}, \frac{l}{D}, S, r\right) \tag{13}$$

Now when $\left(\frac{x}{c}\right) = \left(\frac{x}{c}\right)_0$, $p = p_0$ by definition. Hence from (13), calling θ_0 the value of the function θ when $\left(\frac{x}{c}\right) = \left(\frac{x}{c}\right)_0$,

$$p_0 = \theta_0 \cdot \mu n \tag{14}$$

[3] See for example, Buckingham, Windage Resistance of Steam Turbine Wheels, Bull. Bureau of Standards, **10**: 191–234, 1913; Physically Similar Systems, Phys. Rev., **4**: 345–376, 1914.

Equations (11) and (14) correspond to equations (5) and (6) respectively, and contain the two laws of lubrication in their most general form.

If two bearings are geometrically similar as regards their essential parts, the values of $\frac{c}{D}, \frac{l}{D}$, and the additional ratios r needed for fixing the shape (including the roughness) are the same for both: if the bearings are similarly loaded as regards direction and point of application of the force, the remaining ratios r of equation (11) are the same for both. If, finally, the bearings have the same relative oil supply or are similarly lubricated the value of S is the same for both. Under these conditions equation (11) reduces to

$$f = \Phi\left(\frac{\mu n}{p}\right) \tag{11a}$$

Hence equation (11) states that in geometrically similar bearings which are similarly loaded and lubricated, *the coefficient of friction depends only on the single variable* $\frac{\mu n}{p}$. Equation (14) states that *the carrying power of any bearing is directly proportional to the product of viscosity by revolutions per unit time:* the constant of proportionality

$$\theta_0 = \theta\left[\left(\frac{x}{c}\right)_0, \frac{c}{D}, \frac{l}{D}, S, r\right] \tag{14a}$$

being the same for all geometrically similar bearings which are similarly loaded and lubricated and which are similarly safe, *i.e.* $\left(\frac{x}{c}\right)_0$ = constant.

6. Dynamically similar bearings. Any two geometrically similar bearings B and B' which are similarly loaded and lubricated ($S = S'$) and which are running at the "corresponding" speeds, pressures and viscosities defined by the equation

$$\frac{\mu n}{p} = \frac{\mu' n'}{p'} \tag{15}$$

must, by equations (11) and (12), have the same coefficient of friction and the same relative film thickness. Such bearings may

be termed dynamically similar. The power dissipated in either of them may be calculated from a test made on the others, for by (4)

$$\frac{P}{P'} = \frac{D}{D'} \cdot \frac{n}{n'} \cdot \frac{L}{L'} \tag{16}$$

Moreover by (13) and (15) it follows that $\theta_0 = \theta_0'$, hence by (14)

$$\frac{p_0}{p_0'} = \frac{\mu}{\mu'} \cdot \frac{n}{n'} \tag{17}$$

Thus if the safe load and therefore the carrying power of one bearing has been established experimentally, the carrying power of the other can at once be calculated.

7. Relation of friction to film thickness. This is as far as the problem can be carried by the foregoing general type of reasoning. Further information must be obtained by experiment or by making some assumption as to the geometrical form of the oil film. In the particular limiting case of a perfectly cylindrical bearing free from end-effects and cavitation (i.e., completely lubricated) it is a simple matter to deduce the relation

$$f = \frac{\pi^2 \cdot \frac{D}{c} \cdot \frac{\mu n}{p}}{\sqrt{\frac{x}{c}\left(2 - \frac{x}{c}\right)}} \tag{18}$$

Equation (18) merits careful scrutiny. It is an expression for the coefficient of friction of a bearing constrained to run with a given film thickness. While the coefficient of friction for any given value of $\frac{\mu n}{p}$ is excessively high at very small film thicknesses, it will have fallen to within 16 per cent of its minimum value when the film thickness has become as large as half the clearance. Hence the relation

$$f_c = \pi^2 \cdot \frac{D}{c} \cdot \frac{\mu n}{p} \tag{19}$$

to which (18) reduces when $x = c$, may conveniently be used as an approximate expression for the coefficient of friction, f. In

order to learn just how large $\frac{\mu n}{p}$ must be in order to thicken up the film to any given value we should need to know the complete form of the function ψ in equation (12); but for all thicknesses greater than $\frac{c}{2}$, the value of f_c from (19) may be used for f with an error certainly less than 16 per cent.

III. AN APPROXIMATE TREATMENT OF THERMAL EFFECTS

8. Effect of thermal expansion. This can be shown to be negligible compared with other outstanding uncertainties.

9. Effect of temperature on viscosity. The viscosity-temperature curves of most lubricating oils can be closely fitted by the empirical equation

$$\mu = \frac{\mu_1 T_1}{T} \tag{20}$$

in which T denotes $t - \tau$, t being the temperature of the oil, and τ an empirical constant not greatly different from the solidifying temperature. The subscript designates values at room temperature; thus $T_1 = t_1 - \tau$ and $\mu = \mu_1$ when $t = t_1$.

10. Relation of temperature to speed. From (19) and (20)

$$f_c = \pi^2 \cdot \frac{D}{c} \cdot \frac{\mu_1 n}{p} \cdot \frac{T_1}{T} \tag{21}$$

Assume Newton's law of cooling and let h be the heat carried off in unit time by the air, by the jacket water, or otherwise, per unit temperature elevation above room temperature. Then if J denote the mechanical equivalent of heat, equation (4) leads to the condition for equilibrium

$$\frac{\pi DnLf}{J} = h(t - t_1) = h(T - T_1) \tag{22}$$

At high enough values of $\frac{\mu n}{p}$ that f_c may be written f, we may solve (22) for the relative temperature T getting

$$T = \frac{T_1}{2}(1 + \sqrt{1 + kn^2}) \tag{23}$$

or for the actual permanent running temperature t getting

$$t = \tau + \tfrac{1}{2}(t_1 - \tau)\ (1 + \sqrt{1 + kn^2}) \tag{24}$$

in which the constant k is given by the relation

$$k \equiv \frac{4\pi^3}{Jh} \cdot \frac{\mu_1}{T_1} \cdot \frac{l}{c} \cdot D^3 \tag{25}$$

11. An approximate expression for the coefficient of friction. From (23) and (21), writing f for f_c, approximately

$$f = \pi^2 \cdot \frac{D}{c} \cdot \frac{\mu_1 n}{p} \left(\frac{2}{1 + \sqrt{1 + kn^2}} \right) \tag{26}$$

The coefficient of friction therefore increases less rapidly than the speed, this falling off being more pronounced the larger the value of the heating constant k.

12. An approximate expression for carrying power. Similarly substituting from (23) into (14)

$$p_0 = \theta_0 \cdot \mu_1 n \left(\frac{2}{1 + \sqrt{1 + kn^2}} \right) \tag{27}$$

Carrying power therefore increases less rapidly than speed, approaching asymptotically the limiting value

$$(p_0)_{max} = \frac{2}{\sqrt{k}} \cdot \theta_0 \mu_1 \tag{28}$$

IV. EXPERIMENTS

13. Purpose. The writer made a series of experiments on friction and carrying power at the Massachusetts Institute of Technology in 1909. Their purpose was to determine the influence of viscosity, oil-supply, and other factors on friction, and to test experimentally the notion that carrying power may increase with speed.

14. Apparatus. The journal was of hardened steel running in a brass bearing 3 inches long by 1 inch in diameter, the radial clearance having been 0.002 inch when the bearing was new. Film thickness variations were studied electrically by measuring

the resistance of the oil film. Four oils were used: sperm, lard, ordinary mineral machine oil, and a heavy cylinder oil. The chief source of uncertainty was in the determination of absolute viscosities. The friction observations themselves were relatively free from error. The machine was provided with an ordinary *complete* bearing. The only feature which need be emphasized is that the conditions of practice were thus more closely reproduced than could have been done on any machine with only a half-bearing (as in the Tower type), or in which the two halves of the bearing are *both* forced against the journal by external pressure (as in the Thurston type).

15. Summary of qualitative results. The following results were obtained:

WITH INCREASE OF:	OVER THE RANGE FROM:	THE COEFFICIENT OF FRICTION, f:	THE FILM THICKNESS, x:
speed, n.....	300 to 1500 r.p.m.	increased; $f \propto n$, nearly.	increased; at first slowly, then rapidly, then slowly, approaching a limit.
bearing pressure, p.	40 to 250 lbs. per sq. in.	decreased; $f \propto \frac{1}{p}$ nearly.	decreased; approaching zero asymptotically.
viscosity, μ.....	10^{-7} to 14· 10^{-7} in., min., lb.-wt. units.	increased; $f \propto \mu$, nearly.	increased.
oil feed.....	2 to 60 drops per min.	decreased; at first rapidly, but only slightly after 10 drops per min.	increased; at first rapidly, then slowly.

Any change in viscosity produced the same effect whether the viscosity was altered by changing oils or by changing temperatures. There was no indication of complete "film rupture," the film thickness decreasing continuously with increasing load as far as the observations were carried. The above cited evidence that film thickness increases with speed and viscosity

and decreases with increase in load, leads at once to the conclusion that carrying power increases when speed increases.

16. Relation of friction to $\frac{\mu n}{p}$. The results of about 200 independent determinations of the coefficient of friction of sperm, lard, and machine oils at 10 or more drops per minute may be represented by the equation:

$$f = 0.002 + 6800\frac{\mu n}{p} \tag{29}$$

for values of $10^7 \frac{\mu n}{p}$ between 1 and 40. The average deviation of the observed points from this straight line is about 15 per cent. There is no systematic tendency for the points corresponding to the respective oils to cluster together. Equation (29), then, is a particular form of the general equation (11a) for approximately complete lubrication. It would be applicable to any completely lubricated bearing loaded in the ordinary manner, which is geometrically similar to the experimental bearing in every respect, including the effect of wear. Equation (29) is cited primarily as an experimental verification of the conclusion that in any given bearing with a given oil supply, the coefficient of friction depends only on the single variable $\frac{\mu n}{p}$.

A shorter series of experiments with cylinder oil over the interval from $10^7 \frac{\mu n}{p} = 30$ to 250 led, with the same degree of approximation, to the equation

$$f = 0.015 + 6800\frac{\mu n}{p} \tag{30}$$

The difference in the constant terms of the two equations is doubtless due to the fact that, with this very heavy oil, the bearing did not completely fill itself. If this surmise is correct, we have an illustration of the effect of S in equation (11).

V. GENERAL INFERENCES

17. Relating to the coördination of existing data. Since, by equations (11) and (12), the laws of lubrication may involve, in addition to the variable $\frac{\mu n}{p}$, any of the quantities $\frac{c}{D}, \frac{l}{D}$, S, and the r's, in attempting to coördinate data obtained by different observers we must consider the possible influence of all these factors.

18. Relating to future experiments. (1) In formulating the laws of lubrication it has been seen that μ, n, and p can occur only when combined into the single variable $\frac{\mu n}{p}$; hence the influence of all three of these quantities may be found by experimenting with any one of them. (2) It is legitimate to determine the dynamical and thermal properties of bearings by separate experiments. For example, the laws of friction may be studied under isothermal conditions; while thermal characteristics, such as the constant k of equations (23) to (28), might be determined with a perfectly stationary dummy bearing, in which the heat is generated by a heating coil instead of by friction. (3) The conception of dynamically similar bearings will make it possible to evaluate the constants needed in design by the use of models without waiting for the complete determination of the laws of lubrication.

MINERALOGY.—*Babingtonite from Passaic County, New Jersey.* CLARENCE N. FENNER, Geophysical Laboratory.

In the deposits of zeolitic minerals occurring in the Watchung trap ridges of northern New Jersey certain cavities which are found in abundance evidently testify to the former presence of some mineral which has been removed in solution. These cavities or casts are familiar to many mineralogists who have visited the localities referred to and the nature of the mineral which formerly filled them has been a matter of considerable speculation, but no very satisfactory conclusion has been reached.

The casts are seen most frequently among masses of quartz,

prehnite, datolite, or pectolite. Among other minerals some indication of their presence may at times be found, but often they have been filled with later minerals and their outlines have been more or less obliterated. The mineral which formerly occupied the cavities evidently occurred most frequently in radiating groups, sometimes extending from a centre in various directions, sometimes assuming a rather flattened or fan-like form. A length of 3 or 4 inches is not uncommon, and in one instance the impressions of a group of fan-like crystals was observed, which measured 8½ inches (21.5 cm.) in length and 7½ inches (19 cm.) in greatest breadth. Apparently a considerable portion of the centre of the group had been broken away, so that the original length was probably 3 or 4 inches greater. In cross-section the casts of individual crystals are either lozenge-shaped or nearly rectangular. Both forms are often associated in the same hand-specimen.

In an article[1] which the present writer published several years ago it was shown that the various minerals which are found in these deposits exhibit a well-defined sequence of deposition and that during earlier periods in their history the assemblage of minerals differed greatly from that now found. The earliest period was characterized by the formation of large quantities of quartz, albite, and the mineral represented by the casts, together with a small amount of garnet, hematite, and sulphides. The question of the nature of the mineral which had been present in such abundance and had been removed was considered most interesting and considerable time was given to an attempt to obtain a clue to its nature. In one of the microscopic sections a few small crystals of some unrecognized mineral were found, and in occasional hand-specimens out of several hundred which were gone over some small and almost entirely decomposed remnants of the same substance were discovered. A slight description of the mineral was given and from its general characteristics it was considered to be a rather abnormal amphibole, perhaps

[1] The Watchung Basalt and the Paragenesis of Its Zeolites and Other Secondary Minerals; Annals N. Y. Acad. Sci., XX, 2, Pt. II, 93–187. 1910.

arfvedsonite. Certain slight bits of evidence pointed to the likelihood that this represented the mineral of the casts.

In subsequent visits to these localities a constant watch was kept for further evidence on the matter, but it was not until the past summer (July, 1914) that much of value was found. At that time a visit to the Francisco quarry at Great Notch resulted in the discovery of a larger quantity of the mineral than had been seen at any previous time. The amount was still very small, the largest mass having a greatest dimension of only 20 mm., and the total weight collected was probably little more than a gram. The relations were such, however, as to connect the mineral quite definitely with the casts, and it was possible to make determinations which showed the mineral to be the rare species, babingtonite (Ca, Fe, Mn) $SiO_3 + Fe_2 (SiO_3)_3$.

In several instances the small crystal-remnants found occupy portions of the characteristic cavities of rectangular or lozenge-shaped sections in quartz, or are intergrown in a blade-like form with quartz-crystals. In almost all cases alteration has produced a decomposition-product on the crystal surfaces of a woolly or asbestos-like material, nearly white in color, which obscures the original crystal-faces. The same substance frequently penetrates deeply within the crystals along cracks or cleavage planes. This feature would render it very difficult to pick out pure material from the small quantity of the mineral at hand in sufficient amount for a satisafctory quantitative analysis, and this was not attempted. It was necessary, therefore, to establish its identity by other means. The unaltered material is nearly black and opaque in mass, but translucent and brown or dark-green in thin fragments. The lustre is shining or splendent—nearly that of anthracite.

Through the kindness of Dr. E. T. Wherry of the National Museum, several specimens of babingtonite were loaned from their collection and it was thus made possible to make direct comparisons with undoubted material.

In the determination of properties the material used was either that from Great Notch or that in the microscopic section prepared several years ago, which came from Paterson.

Chemical and physical properties. Qualitative tests on small portions of the mineral gave strong reactions for iron, manganese, lime, and silica. Tests for alumina, titanium oxide, magnesia, and alkalies gave no indication of the presence of these.

Before the blowpipe the mineral fuses without difficulty to a black, slightly magnetic globule. Babingtonite from Baveno, Italy (Nat. Mus. No. 86,183) and from Norway (Nat. Mus. No. 78,559) gave the same test.

There is a very perfect, nearly scaly cleavage, which gives a mirror-like surface. By comparison with the National Museum crystals from Italy and Norway this is seen to be the cleavage parallel to the basal pinacoid *c*, when the crystal is set up in the position which Dauber has chosen.[2]

In the prism zone the cleavage is of columnar or fibrous appearance.

In babingtonite crystals from Italy and Norway vertical striation of faces in the prism zone is prominent. In a rough crystal which was broken out of the aggregate in the New Jersey specimen the same appearance was very evident. Moreover, in the casts similar striations frequently appear.

A determination of specific gravity on apparently unaltered fragments picked out under a binocular, using Rohrbach's solution according to Merwin's method,[3] gave a density of 3.398 at 21°.

The optical properties were found to be as follows: Greatest index of refraction in sodium light (immersion method) = 1.74. Birefringence, $\gamma - \alpha = 0.032$. Optical character biaxial and positive. The axial angle, 2 V, is evidently large, the bar of the interference figure being nearly straight. Hintze gives 2 V = 60°–65°. In the New Jersey mineral a rather larger angle is suggested by the slight curvature of the bar, but babingtonite from Buckland, Mass. (Nat. Mus. No. 80,668), and from Norway

[2] Crystallographers differ in their choice of positions for setting up the crystals. The position chosen by Dana and Hintze brings out the crystallographic similarity to the monoclinic pyroxenes, but the dominant habit of growth of the crystals is prismatic in the direction of Dauber's prism zone. (See article by C. Palache and F. R. Fraprie in Proc. Amer. Acad. Arts and Sci., **38**, 11: 383–393. 1902). Dauber's orientation is adopted in the present paper.

[3] H. E. Merwin, Am. J. Sci., (4) **32**: 425–428. 1911.

(Nat. Mus. No. 78,559) gave the same figure. Dispersion is very noticeable. Pleochroism is remarkably strong. In thin sections α = dark green, β = claret, γ = pale brown. Absorption formula $\alpha > \beta = \gamma$. Extinction angles up to 43° between γ and the basal cleavage were observed. Vogt (quoted by Hintze) gives an angle of 44° for the extinction-direction on sections parallel with *o* (011).

Palache and Fraprie, in the article cited, give for the angle between *b* and *g*, two prominent faces in the prism zone, 64°39′. In the lozenge-shaped casts from New Jersey the faces are usually

	PROPERTIES OF BABINGTONITE AS GIVEN BY VARIOUS AUTHORITIES	PROPERTIES OF BABINGTONITE FROM NEW JERSEY
Composition	(Ca, Fe, Mn) $Sio_3 + Fe_2 (Sio_3)_2$	Qualitative reactions for CaO, Fe, Mn, SiO_2.
Color	Greenish- or brownish-black	Greenish- or brownish-black.
Lustre	Vitreous, splendent	Vitreous, splendent.
Cleavage	Perfect on base, less so on prism 110	Perfect on base, columnar or fibrous on prism zone.
Fracture	Imperfectly conchoidal	Imperfectly conchoidal.
Fusibility	Easily fusible to a black, magnetic globule	Easily fusible to a black, magnetic globule.
Hardness	5.5–6	About 6.
Density	3.35–3.37	3.40.
Index of refraction	n = 1.72 (Winchell)	γ = 1.74.
Birefringence	$\gamma - \alpha$ = 0.032	$\gamma - \alpha$ = 0.032.
Optical character	Biaxial, positive. 2 V = 60° – 65°	Biaxial, positive. 2 V large.
Dispersion	Strong	Strong.
Pleochroism	Intense α = strong emerald green β = pale violet brown γ = deep-brown	Intense. α = dark blue-green. β = claret. γ = pale brown.
Crystal angle between 010 and $2\bar{1}0$	64° 39′ (Palache and Fraprie)	Average of 35 measurements of casts gives 64° 18′.
Crystal angle between 110 and $2\bar{1}0$	89° 18′ (Palache and Fraprie)	Casts nearly rectangular.
Extinction angle	44° with basal cleavage	43° with basal cleavage.

slightly irregular, but by making a large number of measurements under the microscope, the average should give a close approximation to the true value. Thirty-five measurements of the acute angle were made, and the average was 64°18′.

In Palache and Fraprie's measurements the angle between *g* and *h*, the latter being a prismatic face of less prominent development in their crystals, was measured as 89°18′. This probably corresponds to the casts of nearly rectangular cross-section which were often observed. No measurement of this angle was attempted, as thin laminae of quartz frequently project from the face corresponding to *h*.

The results of various tests on the New Jersey mineral are tabulated above in comparison with the properties of babingtonite. From this table it is evident that the mineral under discussion agrees perfectly in all important respects with babingtonite.

As a further confirmation etching-tests with hydrofluoric acid were made upon cleavage-flakes (parallel with the base *c*) of this mineral and of babingtonite from Baveno, Italy (Nat. Mus. No. 86,183). The two were placed together in a platinum basket and suspended for five seconds in boiling commercial hydrofluoric acid. With both specimens the etch-pits were very minute and required careful observation with a high-power objective to be satisfactorily studied, but, so far as could be determined, the shape and size were the same in the two instances. Figure 1 shows their character.

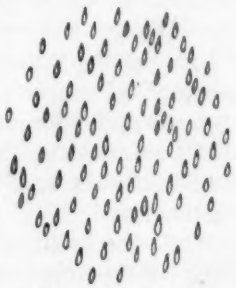

Fig. 1. Etch-figures on babingtonite, produced by action of hydrofluoric acid. Magnification 250 ×.

Hintze and Dana cite but few localities where babingtonite has been found and it may be classed among the rarer minerals. The crystals are generally of minute size. Its usual occurrence appears to be in granite, syenite, gneiss, etc., apparently not generally as an original igneous constituent, but associated with such minerals as epidote, garnet, alkali feldspars, and quartz, in cavities. Palache and Fraprie[4] describe its occurrence at Somerville, Mass., where it is found "in veins and pockets composed chiefly of prehnite, which traverse a large dyke of diabase." Quartz, epidote, feldspar, laumontite, stilbite, chabazite and other

[4] C. Palache and F. R. Fraprie, loc. cit.

minerals are associated with it. The general relations appear quite similar to those in New Jersey.

It seems quite remarkable that in New Jersey it should have been formed in such abundance and in crystals of such great size, and that later it should have been almost totally removed. Its disappearance cannot well be ascribed to weathering, for in many instances the casts have been filled with minerals of an early period of deposition. The formation and subsequent removal of the mineral indicate that for it the conditions of chemical stability were satisfied for a brief period only during the processes of general mineral deposition.

ZOOLOGY.—*The correlation between the bathymetrical and the geographical range in the recent crinoids.* AUSTIN H. CLARK, National Museum.

In a paper published about a year ago[1] I remarked that the geographical range of a crinoid species, genus or higher group is approximately proportionate to its bathymetric range, but at that time I did not have the facts upon which I based the deduction in such form as to be able to present them in a convincing manner.

While the bathymetric range of any type can mean but one thing—the number of fathoms (or meters) between the highest and the lowest limit of the zone in which it occurs—the geographical range may be interpreted in two different ways: (1) as the actual area, calculated as the sum of the geographical units within which the type is actually known to occur, or (2) as the area in which the type potentially occurs, that is, the area over which physical conditions are such as to suggest that, if not found at all points within it now, it has or may have, occurred at all points within it at some time in the not remote past.

According to the first method a count is made of the units of area within which the type has actually been taken; the figures are therefore of very varying value for different groups, for many types, undoubtedly with an enormous geographical range, have been taken at only a very few widely scattered localities, while

[1] Internationale Revue der gesamten Hydrobiologie und Hydrographie, Bd. 6, Heft 1, S. 29.

others, with a much more restricted geographical range, have been reported from all points within a very large area.

The second method first of all calls for the determination of some restricted region as the center of distribution, for it makes considerable difference in calculating the potential geographical range of a group like the Thysanometrinæ, occurring from southern Japan southward to the Admiralty Islands, and again in the Caribbean Sea, whether we assume the center of distribution to be the Malayan region, and therefore that the connection between the two localities is westward from the line between southern Japan and the Admiralty Islands, or whether we assume some other center of distribution, and a connection eastward through Oceania and over Central America.

I have elsewhere[2] given my reasons for considering the Malayan region as the center of distribution for the recent (and later fossil) crinoids, explaining the similarity between the fauna of southern Japan and Hawaii and the Caribbean Islands as the result of their present similar isolation from the Malayan region, an isolation which has permitted the persistence on the periphery of the area inhabited by the crinoids as a whole of types which in all the intermediate regions have been extirpated by more efficient competitors of subsequent origin.

If we divide the map of the world into areas measuring 15° on each side, we find that the number of such divisions covering the geographic ranges of each of the families of recent crinoids is as follows:

Capillasterinæ	42	Thysanometrinæ	5
Comactiniinæ	28	Zenometrinæ	19
Comasterinæ	30	Perometrinæ	12
Zygometridæ	12	Heliometrinæ	68
Himerometridæ	22	Bathymetrinæ	12
Stephanometridæ	14	Pentametrocrinidæ	13
Mariametridæ	30	Atelecrinidæ	7
Colobometridæ	31	Pentacrinitida	20
Tropiometridæ	33	Apiocrinidæ	2
Calometridæ	10	Phrynocrinidæ	2
Ptilometrinæ	10	Bourgueticrinidæ	20
Thalassometrinæ	36	Holopodidæ	2
Charitometridæ	18	Plicatocrinidæ	14
Antedoninæ	43		

[2] Internationale Revue der gesamten Hydrobiologie und Hydrographie, Bd. 6, Heft 1, S. 24.

Plotting these together with the maximum range for each family (fig. 1), we notice very little similarity between the two lines. It is interesting to observe, however, that in the left half of the diagram, including all of the better known families, the agreement between the geographical and the bathymetrical ranges

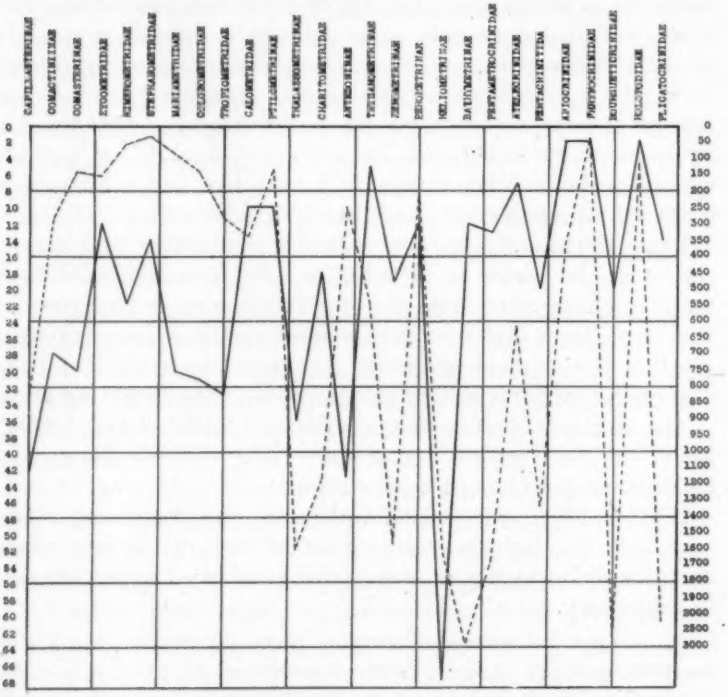

Fig. 1. Comparison between the maximum bathymetric range (. . . .) and the geographical range, expressed as the sum of the areas of 15° on each side within which the families occur (———).

is somewhat closer than in the right half, which includes the rarer types.

There is a very grave source of error in comparing the geographical and the bathymetrical ranges of any animal group by this method, and that is that single observations are always of far

greater value from a bathymetric than from a geographic standpoint, so that, broadly speaking, our knowledge in regard to the bathymetric distribution of animal types is much more detailed than our knowledge of the limits of the geographical range of the same types.

The bathymetric range is measured on a single line assumed to extend perpendicularly downward from the surface to the deepest part of the sea. But the points on this line are determined from observations everywhere. That is, the entire volume, or cubical content, of the ocean basins furnishes data projected upon a single line. For example, a certain type occurs in Alaska in 5 fathoms, in the Crozet Islands in 1600 fathoms, and off Greenland in 300 fathoms; its bathymetric range is, therefore, from 5 to 1600, or 1595, fathoms.

Thus our knowledge of the bathymetric ranges of the larger groups is reasonably complete, as a result of this method of reducing to terms of a single dimension records which, strictly speaking, are taken in three dimensions.

With the geographical ranges calculated as the sum of all the areas measuring 15° on each side within which a given type occurs the case is very different. Investigation has largely been localized within certain restricted areas which, for some reason or other, have proved to be especially interesting, or where exceptional opportunities for study have been presented. Within these areas there are very numerous records all of which, though marking different steps in, and increasing our knowledge of, the bathymetic scale, fall in the same geographical unit, and hence are the geographical equivalent of only a single record.

Intensive study of any one locality increases our knowledge of the bathymetric distribution of all the endemic types, while adding nothing to our knowledge of the geographical distribution of the same types; similarly future investigations in many of the now little known regions of the world will greatly increase our knowledge of the geographical range of many groups, at the same time giving us nothing new in reference to their bathymetrical range.

Thus it is evident that the comparison of the bathymetric ranges of the families of recent crinoids, which for the most

part are known to within a comparatively small limit of error, with the geographical ranges calculated as the sum of the units measuring 15° on each side within which they occur, though apparently perfectly logical, at the present state of our knowledge is not practicable, and cannot give results of value.

If we assume that the Malayan region is the center of distribution of the recent crinoids, a very simple way of expressing the comparative potential geographical range of the several crinoid types becomes possible. If we multiply the number of units of 15° between the meridians bounding the region inhabited by each type by the number of units of 15° between the parallels of latitude bounding the same regions, always reckoning east and west and north and south from the Malayan region, we will obtain for all types (excepting only the Holopodidæ which, alone among the crinoid families, does not occur in the Malayan region) strictly comparable areas expressed in units of 15° on each side, that is, including 225 "square" degrees each.

The geographical ranges of the families of recent crinoids expressed in these units are:

Capillasterinæ	30 × 6 = 180	Thysanometrinæ	18 × 4 = 72
Comactiniinæ	30 × 6 = 180	Zenometrinæ	24 × 10 = 240
Comasterinæ	12 × 6 = 72	Perometrinæ	17 × 4 = 68
Zygometridæ	6 × 5 = 30	Heliometrinæ	24 × 12 = 288
Himerometridæ	10 × 5 = 50	Bathymetrinæ	24 × 10 = 240
Stephanometridæ	9 × 4 = 36	Pentametrocrinidæ	16 × 10 = 160
Mariametridæ	12 × 6 = 72	Atelecrinidæ	20 × 3 = 60
Colobometridæ	20 × 6 = 120	Pentacrinitida	20 × 6 = 120
Tropiometridæ	19 × 6 = 114	Apiocrinidæ	1 × 2 = 2
Calometridæ	5 × 6 = 30	Phrynocrinidæ	6 × 2 = 12
Ptilometrinæ	5 × 5 = 25	Bourgueticrinidæ	16 × 10 = 160
Thalassometrinæ	24 × 8 = 192	Holopodidæ	2 × 2 = 4
Charitometridæ	20 × 6 = 120	Plicatocrinidæ	24 × 9 = 216
Antedoninæ	18 × 6 = 108		

On the accompanying diagram (fig. 2) are shown the maximum bathymetric range of each family of recent crinoids (in a dotted line) and the geographical range calculated according to the method just described (in an unbroken line).

The correspondence between the bathymetric range and the geographical range as thus calculated is very striking, bringing

our very forcibly the fact that the potential geographical range of a given group is proportionate to its bathymetrical range.

It will be noticed that, in the comparison between the bathymetrical and the geographical ranges as given on the diagram, 3000 fathoms in depth is the equivalent of 256 × 225, or 6400, square

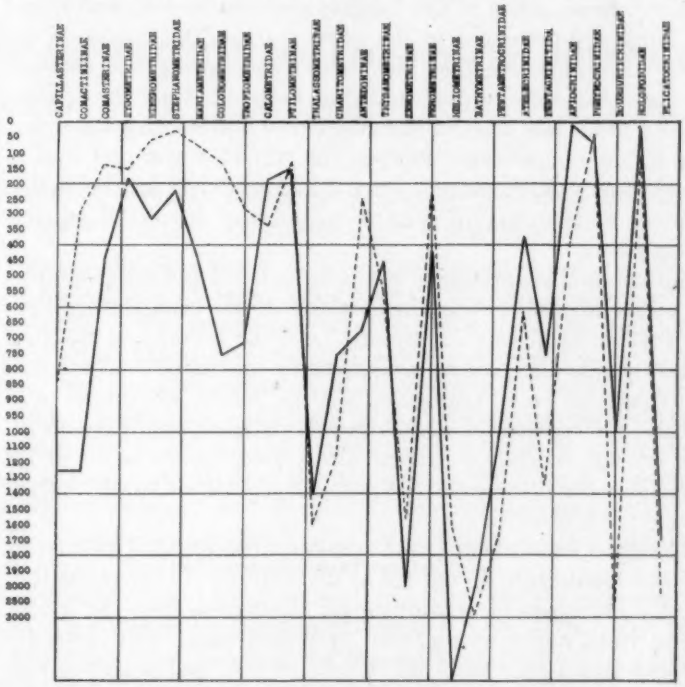

Fig. 2. Comparison between the maximum bathymetric range (. . . . .) and the geographical range expressed, in units of 15° square (225 square degrees), as the product of the number of degrees on the equator between the meridians passing through the limits of the distribution West and East of the Malayan region, times the number of degrees between the Parallels of Latitude bounding the area of occurrence of the Families of Recent Crinoids (———).

degrees; therefore every fathom of increase in the bathymetric range of a given type implies an increase of approximately 2.13 square degrees in the geographical range, and every increase of

100 fathoms in the bathymetric range is correlated with an increase of 213.33 square degrees in the geographical range.

In the families confined to comparatively warm water the geographical range is somewhat greater than the bathymetrical range, when 100 fathoms is considered as the equivalent of 213.33 square degrees, while in the families confined to cold water, and the families represented in the polar regions, the reverse is the case. This indicates that the curve representing the decrease in area of the units measuring 15° on each side from the equator to the poles is less marked than the curve representing the difference in the temperature between the surface water and that of the abysses (which plays a very important part in the bathymetrical distribution of marine organisms) from the equator to the poles.

This method of comparing the bathymetrical and the geographical range of marine organisms which at no time during their developmental history are pelagic takes no account of the land masses within the geographical areas as calculated. These land masses appear to be negligible; in other words, we appear to be justified in considering them as everywhere potentially habitable by the crinoids occurring along their shores as far as the meridians of longitude and the parallels of latitude to which these crinoids now extend.

The great tropical currents flowing northward, the Gulf Stream and the Kuro-Siwo, do not act as distributors of crinoids as they do of other types of organisms, for the reason that the littoral forms which might be supposed to extend their range along the shores washed by them, but which are confined within a very limited range of temperature and of salinity, cannot survive the conditions in the winter, when the currents in the northernmost part of their course move southward and off shore, and when further southward their inshore border is chilled and freshened by drainage from the land.

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